Where a business has two competing investment opportunities the one with the higher NPV should be selected.

Logically the value of a business should be the sum of all of the projects which it has in operation at the particular time.

Q

ACTIVITY 3: QUESTION

What is the NPV of an investment opportunity which involves an immediate outflow of £260,000, which will give rise to an inflow of £300,000 after one year, assuming an interest rate of 8%?

A

ACTIVITY 3: ANSWER

$$NPV = -260{,}000 + \frac{300{,}000}{1+\left(\dfrac{8}{100}\right)}$$

$$= -260{,}000 + 277{,}778$$

$$= £17{,}778$$

All other things being equal, the investment should be undertaken.

Q

ACTIVITY 4: QUESTION

What is the NPV of an opportunity which involves *receiving* £10m immediately and *paying* £12m after one year assuming a 12% p.a. interest rate?

A

ACTIVITY 4: ANSWER

$$NPV = £10m + \frac{£12m}{1+\left(\dfrac{12}{100}\right)}$$

$$= £10m - £10.7$$

$$= -£0.7m$$

The ‘investment’ should not be undertaken. Note that this ‘discounting’ approach is just as valid irrespective of whether the future cash flow is a positive or a negative one.

Investment Opportunities Lasting for More than One Year

In reality, few investment opportunities last for only one year. Suppose that an investment opportunity involves an immediate cash outflow C_0 which will give rise to inflows C_1 and C_2 after one and two years respectively. We already know that the present value of C_1(effect on the value of the business), is $C_1/(1 + i)$. By the same logic as we derived this, an amount, say Y, which would grow with interest compounded annually to C_2.

Y invested now will grow to $Y + Yi$ or $Y(1 + i)$ after one year. During the second year this amount will be re-invested and it will then grow to $Y(1+i)+(Y(1+i)\times i)$. This equals

$$C_2 = Y + Yi + (Y + Yi)i$$

(i.e. the original borrowing plus interest on that amount for the first year plus interest on these two during the second year).

This expression expands to:

$$C_2 = Y + Yi + Yi + Yi^2$$

taking Y outside brackets:

$$C_2 = Y(1 + 2i + i^2)$$
$$C_2 = Y(1 + i)^2$$
$$\text{and } Y = C2/(1 + i)^2$$

Following this logic, it can be shown that the present value of any amount of cash receivable after n years (C_n) would be $C_n/(1 + i)^n$.

If we wish to define NPV in a formal, mathematical way, we can say that the NPV of any investment opportunity is given by:

$$NPV = \sum_{t=0}^{n=t} \frac{C_n}{(1+i)^n}$$

(where t is the life of the opportunity in years)

If we wish to put it in plain English, we can say that the NPV of an opportunity is the sum of all of the cash flows associated with it, each one discounted according to how far into the future it one will occur.

Example 2

Antonio plc has estimated that it could achieve cost savings by investing in either of two automatic machines. The accountant's estimate of the actual savings and of the cost of the machines is as follows:

		Type Alpha	Type Beta
		£	£
Cost of the machine (payable now)		50,000	60,000
Estimated annual savings:			
	Year 1	16,000	16,000
	2	12,000	16,000
	3	12,000	16,000
	4	10,000	14,000
	5	10,000	10,000

Which, if either, of these two machines should be bought if the relevant interest rate is 10% per annum throughout?

Note that the savings listed above are not actual inflows (receipts) of cash. They are however 'opportunity' cash inflows. As such, we should recall from Unit 2, they are highly relevant to the present decision.

The NPV of each machine is as follows:

	Type Alpha £	Type Beta £
Present value of cash flows:		
Now (Year 0)	(50,000)	(60,000)
1	$\frac{16{,}000}{(1+0.10)} = 14{,}545$	$\frac{16{,}000}{(1+0.10)} = 14{,}545$
2	$\frac{12{,}000}{(1+0.10)^2} = 9{,}917$	$\frac{16{,}000}{(1+0.10)^2} = 13{,}223$
3	$\frac{12{,}000}{(1+0.10)^3} = 9{,}016$	$\frac{16{,}000}{(1+0.10)^3} = 12{,}021$
4	$\frac{10{,}000}{(1+0.10)^4} = 6{,}830$	$\frac{14{,}000}{(1+0.10)^4} = 9{,}562$
5	$\frac{10{,}000}{(1+0.10)^5} = 6{,}209$	$\frac{10{,}000}{(1+0.10)^5} = 6{,}209$
	£ (3,484)	**£ (4,440)**

This process of converting future cash flows to their present value is known as *discounting*. Tables are readily available which give the factor $1/(1 + i)^n$ for a range of values of i and n. Such a table appears on the very last page of this unit. However, it is not really worth bothering to use a table when it is just as easy to use a calculator to discount from first principles.

It can be seen that both machines have a negative NPV. To purchase either machine would reduce the value of the business. Neither of these two machines should, therefore, be bought on the basis of these estimates of costs (including the relevant interest rate) and of the benefits.

Q

ACTIVITY 5: QUESTION

The supplier of the Alpha type machine has told Antonio plc that it might be prepared to reduce the price of the machine. On the basis of the above calculations, what is the maximum price that Antonio plc should logically be prepared to pay for a machine?

A

ACTIVITY 5: ANSWER

The maximum price which the business should be prepared to pay is the value of the savings which would be expected to result from buying the machine; it would be illogical to pay more for an economic asset than it is worth. To do so would reduce the value of the business, which is precisely the opposite of what businesses seek to do. On the basis of the above calculations, an Alpha type machine is worth £46,517 (i.e. the sum of the present values of the savings). It may be to another business, or even to Antonio plc under other circumstances, the machine may be worth more or less than this. You should note that the value of any economic asset should logically be deduced in this way.

Advantages of Using the NPV Approach

We can conclude that NPV is a valuable means of assessing investment opportunities because:

- It is directly derived from, what appears to be the main financial objective of private sector organisations
- It seems to take account of all the relevant, measurable information about an investment opportunity. This includes the timing of the estimated cash flows and the cost of financing the investment
- It gives clear unambiguous results, which are simple and logical to interpret.

Discounting and Impounding

We have seen that the major objective of discounting future cash flows is to express all cash flows, irrespective of when in the future they are estimated to occur, in the same terms so that a comparison can be made. (Remember our original problem in Example 1 was trying to compare £1,000 to be spent immediately with £1,500 to be received a year later).

An alternative to discounting, i.e. expressing all cash flows in present value terms, is to *compound*, ie to express all cash flows in terms of their value at some specific date in the future. If, as we previously did in respect of Example 1, we assume an interest rate of 10%, we should compound the present cash flow by adding 10% interest to it and then compare it with the unadjusted future cash flow.